VIU VANCOUVER ISLAND UNIVERSITY EXPLORE. DISCOVER. EXCEL.

ABE Math Review Package

This material is intended as a review of skills you once learned and wish to review before your assessment.
Before studying Algebra, you should be familiar with all of the topics included here. Read the explanations carefully, and do as many of each type of question in each exercise until you have mastered the material.

Section One: Check your answers in the answer key at the end of each topic in section one.
Section Two: Check your answers in the answer key on pages 55 - 56.

Do not use a calculator, unless you are instructed to.

Students wishing to take placement tests for math 030 should complete section one. Students wishing to take placement tests for math 045, math 046 and math 047 should complete section one and two.

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Whole Numbers Review

Before studying fractions, you should be familiar with all operations on whole numbers. This work will provide a quick review. Answers are at the end on p.4.

1. ADDITION

In addition, remember to begin with the right hand column, and work to the left.

e.g. 42**3**
+ 13**4**
55**7**

Try these:

(a) 19 + 2

(b) 345 +123

(c) 951 +111

Remember that when the total of any column is greater than 9, you must carry the left digit in that total to the column to the left.

e.g. 43**7** (carry 1)
+12**6**
56**3**

Here is some practice:

(d) 489 + 96

(e) 2754 + 1666

(f) 238 + 777

(g) 1069 + 888

2. SUBTRACTION

Remember to begin at the right hand column and move column by column to the left.

e.g. 76**8**
– 1**5**
75**3**

Do these:

(a) 1349 – 36

(b) 864 – 333

(c) 3834 – 2222

Remember that you should check the accuracy of your answer by adding it to what you took away. If it is correct, your result should be the number you began with. In the example above, 753 + 15 = 768, so we can assume that it is correct.

Remember that when a digit in the bottom number is too big to subtract from the digit in the top number, you borrow from the next column in the top number first.

```
        3 1
e.g.    43
      – 19
        24
```

You can not subtract 9 from 3, so you borrow 1 ten from the tens column first

1. Cross out the 4 in the tens place, and replace it with 3.
2. Borrow 10 from 4 to make 13 in the ones column.
3. Subtract 9 from 13
4. Subtract 1 from 3.

Try these:

```
(d)  753     (e)  70     (f)  8104    (g)  7512    (h)  1000    (i)  3084
   – 409        – 18        – 1987       –  943       –  369       – 2294
```

3. MULTIPLICATION

Take the time to learn your multiplication tables. That will save you a lot of time later on!

Multiply each digit in the top number by the bottom number.
Write the answer from right to left, starting in the ones column.

```
e.g.   12        41
      x 4      x  9
       48       369
```

Remember that any number multiplied by 0 is 0.

Try these:

```
(a)  430          (b)  702          (c)  3011
   x   3             x  41             x    7
```

When you multiply by a two or three digit number, be sure to begin your answer under the ones column. Then continue on the line below the tens and then the hundreds. Always multiply from right to left.
Leave a space under the bottom digit that has already been multiplied OR add a zero in that space to keep the columns lined up neatly and correctly.

Carrying in multiplication is like carrying in addition. Multiply first and then add the number being carried. Line up the digits carefully under the correct column.

```
e.g.     23          638          405
       x 22        x  51        x 266
         46          638         2430
        46_        31900        24300
        506        32538        81000
                               107730
```

Now try these:

(d) $\begin{array}{r} 789 \\ \underline{\times \ \ 46} \end{array}$

(e) $\begin{array}{r} 5106 \\ \underline{\times \ 203} \end{array}$

> When you multiply by 10, 100, 1000, etc,
> add the same number of zeroes to the right of the number.

e.g. 35 x 10 = 350   41 x 100 = 4100   60 x 1000 = 60 000   Tricky!

4. DIVISION

You will need your multiplication tables here too! Learn them well!

Division is the opposite of multiplication. That helps you to check your division answer.

Pay careful attention to lining up digits, as you did in multiplying large numbers, so that you can keep your working straight.

Here are the words: 168 (the **dividend**) divided by 7 (the **divisor**) is 24 (the **quotient**) **OR** 7 into 168 is 24.

e.g.

$$\begin{array}{r} 24 \\ 7\overline{)168} \\ \underline{-14} \\ 28 \\ \underline{-28} \\ 0 \end{array}$$

Step 1: Divide the divisor 7 into 16 = 2. Place the 2 above the 6.
Step 2: Multiply 7 x 2 = 14. Place the 14 under the 16.
Step 3: Subtract: 16 – 14 = 2.
Step 4: Bring down the next number to the right = 8
Step 5: The new number is 28.
Step 6: Divide 7 into 28 = 4. Multiply 4 by the divisor 7 and the answer is 28.
Step 7: Subtract 28 from 28 = 0. The division is complete.

> Check: In division, it is easy to check the answer by multiplying your answer (the **quotient**) by the number you are dividing by (the **divisor**).
> 24 x 7 = 168 so the answer is likely to be correct.

Sometimes there is an amount left over, which is called the **remainder**.
It is placed on the top line with the letter **r** for **remainder** and is part of the **quotient**.

e.g.

$$\begin{array}{r} 569 \text{ r } 3 \\ 6\overline{)3417} \\ \underline{-30} \\ 41 \\ \underline{-36} \\ 57 \\ \underline{-54} \\ 3 \end{array}$$

Try these:

(a) $4\overline{)1631}$ (b) $7\overline{)1046}$ (c) $9\overline{)3004}$

When you check your answer for a division problem with a remainder, multiply your answer (the **quotient**) by the number you are dividing by (the **divisor**) and **add** the **remainder**.

When you divide by a double or triple digit number, you need to use another different skill. You need to **estimate**, which is a process of thoughtful guessing. This takes time and practice.

e.g.

$$\begin{array}{r} 223 \\ 28\overline{)6244} \\ \underline{-56} \\ 64 \\ \underline{-56} \\ 84 \\ \underline{-84} \\ 0 \end{array}$$

Step 1: Think of *about* how many times 28 goes into 6244.
To do that, **round** 28 to 30 (28 ≈ 30) and divide that into 62.
The answer is **approximately or close to 2**.
Step 2: Place 2 above the last digit of 62.
Step 3: 2 x 28 (the divisor) = 56. Then 62 – 56 = 6.
Step 4: Bring down 4. **Estimate** how many times 28, ≈ 30, divides into 64 ≈ 2. Place 2 on the top line.
Step 5: 2 x 28 = 56. 64 – 56 = 8.
Step 6: Bring down 4. **Estimate** how many times 28, ≈ 30, divides into 84 ≈ 3.

Sometimes your **estimate** is not quite right, but it should only be 1 more or 1 less than the correct number, so the **estimate** gives a good starting point. Here is an example with a **remainder**:

e.g.

$$\begin{array}{r} 337 \text{ r } 32 \\ 41\overline{)13849} \\ \underline{-123} \\ 154 \\ \underline{-123} \\ 319 \\ \underline{-287} \\ 32 \end{array}$$

Step 1: **Round** 41 ≈ 40. Divide 40 into the first part of the number.
Step 2: 40 goes into 138 about 3 times. Place 3 above the last digit of 138.
Step 3: 3 x 41 = 123. Subtract 123 from 138 = 15.
Step 4: Bring down 4. Estimate how many times 41 ≈ 40 goes into 154 ≈ 3.
Step 5: Multiply 3 x 41 = 123. 123 from 154 = 31.
Step 6: Bring down 9. Estimate how many times 41 ≈ 40 goes into 319 ≈ 7. 7 x 41 = 287.
Step 7: 319 – 287 = 32. Divide 41 into 32. It cannot divide, so 32 is the remainder.

Try these:

(d) $57\overline{)3477}$ (e) $36\overline{)7601}$ (f) $67\overline{)13467}$ (g) $29\overline{)13796}$

ANSWERS

Addition:	(a) 21	(b) 468	(c) 1062	(d) 585	(e) 4420	(f) 1015	(g) 1957
Subtraction:	(a) 1313	(b) 531	(c) 1612	(d) 344	(e) 52	(f) 6117	(g) 6569
	(h) 631	(i) 790					
Multiplication:	(a) 1290	(b) 28782	(c) 21077	(d) 36294	(e) 1036518		
Division:	(a) 407 r 3	(b) 149 r 3	(c) 333 r 7	(d) 61	(e) 211 r 5	(f) 201	(g) 475 r 21

DECIMAL REVIEW

A. INTRODUCTION TO THE DECIMAL SYSTEM

The Decimal System is another way of expressing a part of a whole number. A decimal is simply a fraction with a denominator of 10, 100, 1 000 or 10 000 etc. The number of decimal places refers to how many zeros will be in the denominator. Note that the number 5.62 is read as five point six two.

The first decimal place refers to tenths $2.3 = 2\frac{3}{10}$

The second decimal place refers to hundredths $2.31 = 2\frac{31}{100}$

The third decimal place refers to thousandths $2.319 = 2\frac{319}{1000}$

Similarly, six decimal places would be a fraction with a denominator of 1 000 000 (millionths). The most common usage of decimals is in our monetary system where 100 cents (2 decimal places) make up one dollar. For example, \$2.41 is really two dollars and forty-one hundredths $(\frac{41}{100})$ of a dollar.

Examples:

1. **Change 2.30 to a fraction**
   Notice that 2.30 is the same as 2.3
   In fact, 2.30 = 2.300 = 2.3000 etc.
   $2.30 = 2\frac{30}{100} = 2\frac{3}{10}$

2. **Change 0.791 to a fraction**
   Notice that 0.791 = .791
   The zero in front of the decimal place is not needed.
   $0.791 = \frac{791}{1000}$

3. **Change .003 to a fraction**
   Notice that the zeros in this example are important.
   $.003 = \frac{3}{1000}$

4. **Simplify 0.0024000**
   Notice that zero at the end or zero as a whole number (to the left of the decimal) is not needed.
   $0.0024000 = .0024$

B. OPERATIONS WITH DECIMALS

1. Addition:

Add $2.50 and $1.35

Place numbers in columns, so that the decimal places are in line. Place the decimal point in the same line for the answer.
Now, add as if adding whole numbers.

```
  $2.50
+ $1.35
  $3.85
```

Add 1.3928 and 12.43 and .412

Add zeros to fill in columns. This will not change the value of the number (see examples on previous page.)

```
   1.3928
+ 12.4300
+  0.4120
  14.2348
```

2. Subtraction:

Subtract $1.30 from $5.45

Set up columns as in addition, and subtract as if subtracting whole numbers.

```
  $5.45
- $1.30
  $4.15
```

Calculate 5 minus .2982

Fill in columns with zeros, as in addition.
Note the difference in wording in these two examples.

```
  5.0000
- 0.2982
  4.7018
```

3. Multiplication:

Multiply 2.12 by 4.2

At first, ignore the decimals, and multiply as if calculating 212 x 42.
Now, add up the decimal places in both numbers and your answer will have that total number of decimal places.

```
2.12   (2 places)
x 4.2  (1 place)
  424
8480
8.904  (3 places)
```

Product of 0.0941 and .02

Multiply. Add enough zeros to show the correct number of decimal places

```
  .0941     (4 places)
  x .02     (2 places)
.001882     (total = 6)
```

Multiply .5624 by 1000

Notice .5624 x 1000 = 562.4, so multiplying x 1000 (0 decimal places) is the same as moving the decimal place 3 places (since 1000 has 3 zeros) to the right.

.5624	(4 places)
x 1000	(0 places)
562.4000	(total = 4)
or 562.4	

Similarly,
.58 x 10 = 5.8   5.636 x 10 000 = 56 360
.58 x .1 = .058   .58 x .001 = .00058

Division: Divide 2.322 by .12

The divisor (.12) must be changed to a whole number (12) by moving the decimal point 2 places to the right, in both numbers. In the answer, place the decimal point directly above the numbers. Note that zeros must be added to complete the division.

$$.12\overline{)2.322}$$

$$
\begin{array}{r}
19.35 \\
12\overline{)232.20} \\
\underline{12} \\
112 \\
\underline{108} \\
42 \\
\underline{36} \\
60 \\
\underline{60} \\
0
\end{array}
$$

Divide .003 into 51

Firstly, notice the difference in wording in these two division questions. Zero must be added to the number 51 in order to move the decimal 3 places to the right.

$$.003\overline{)51}$$

$$
\begin{array}{r}
17000. \\
3\overline{)51000.} \\
\underline{3} \\
21 \\
\underline{21} \\
0
\end{array}
$$

Divide 254.25 by 1000

Notice 254.25 ÷ 1000 = .25425. So, dividing by 1000 is the same as moving the decimal 3 places to the left (since 1000 has 3 zeros).

$$
\begin{array}{l}
\qquad\quad 0.25425 \\
1000\overline{)254.250} \\
\qquad \underline{200\ 0} \\
\qquad\ \ 54\ 25 \\
\qquad\ \ \underline{50\ 00} \\
\qquad\quad 4\ 250 \\
\qquad\quad \underline{4\ 000} \\
\qquad\qquad 2500 \\
\qquad\qquad \underline{2000} \\
\qquad\qquad\ \ 5000 \\
\qquad\qquad\ \ \underline{5000} \\
\qquad\qquad\qquad\ 0
\end{array}
$$

Similarly, $32.52 \div 10 = 3.252$

$2.6 \div 10\,000 = .00026$

EXERCISE 1: Decimal Operations Do not use a calculator.

a) Change to fractions: (remember to reduce)

1) 5.8 **2)** 27.3400 **3)** 30.02 **4)** 0.590 **5)** 3.075

b) Addition:

1) 2.49 + .32 **2)** 0.042 plus .00982 **3)** 7.342 and 2 and 7.65

4) 1743.2 + 2.984 + 12.35 **5)** 2.76 more than 8.4590

c) Subtraction:

1) 2.036 from 4.478 **2)** 12.258 from 13 **3)** 670.1 minus 589.213

4) 0.1002 minus 0.05 **5)** 19.6 decreased by 5.349

d) Multiplication:

1) .21 by .04 **2)** .42 x .218 **3)** .75 times 132.786

4) Product of .009 and 2.003 **5)** .25 of 288

6) 9.4325 by 1000 **7)** 9.4325 by .001

e) Division:

1) 248 divided by 0.8 **2)** 15.47 divided by .091

3) 40.4 into 828.2 **4)** 0.0338 divided by 1.30

5) .0025 into 1.875 **6)** 923.56 divided by 1000

7) 923.56 divided by .01

C. CONVERSION AND ROUNDING OFF

1. Converting Decimals to Fractions:

When changing decimals to fractions, simply create a fraction with 10, 100, 1000 etc. in the denominator. The number of zeros in the denominator is the same as the number of decimal places.

$$0.913 = \frac{913}{1000}$$

$$5.25 = 5\frac{25}{100} = 5\frac{1}{4}$$

2. Converting Fractions to Decimals:

When changing a fraction to a decimal, simply divide the denominator into the numerator. Every time a fraction is changed to a decimal, the division will either stop (as in $\frac{4}{5}$ = 0.8) or the division will go on forever by repeating (as in $\frac{2}{3} = .\dot{6}$). Notice that a repeating decimal is shown by a dot over the number (if only one number repeats) or as a bar (if more than one number repeats, as in $\frac{2}{11} = .\overline{18}$)

$$\frac{4}{5} = 4 \div 5 = 0.8$$

$$\frac{2}{3} = 2 \div 3 = 0.666\ldots = .\dot{6}$$

$$\frac{2}{11} = 2 \div 11 = .1818\ldots = .\overline{18}$$

3. Repeating Decimals

Following is a list of some repeating decimals:

$\frac{1}{3} = .\dot{3}$ $\quad \frac{1}{6} = .1\dot{6}$ $\quad \frac{1}{7} = .\overline{142857}$ $\quad \frac{2}{3} = .\dot{6}$ $\quad \frac{5}{6} = .8\dot{3}$

$\frac{1}{9} = .\overline{11}$ or $.\dot{1}$ $\quad \frac{2}{9} = .\overline{22}$ or $.\dot{2}$ $\quad \frac{4}{9} = .\overline{44}$ or $.\dot{4}$

$\frac{5}{9} = .\overline{55}$ or $.\dot{5}$ $\quad \frac{7}{9} = .\overline{77}$ or $.\dot{7}$ $\quad \frac{8}{9} = .\overline{88}$ or $.\dot{8}$

4. Rounding off:

If we want to divide \$2.00 into 3 equal parts, we would want our answer to be to the nearest cent (or nearest hundredth). Since our answer is closer to 67 cents than 66 cents, we would round off our answer to \$0.67.

$$3\overline{)2.000} = 0.666\ldots$$

When rounding to the nearest thousandth, we

2.6549 to nearest thousandth = 2.655

want our answer to have 3 decimal places. If the fourth decimal place has a 5 or greater, round up. If less than a 5, do not round up.

2.6549 to nearest hundredth = 2.65
3.95 to nearest tenth = 4.0

EXERCISE 2: Conversion and Rounding Off

a) Change to Fractions: (Remember to reduce whenever possible)

1. 2.591 **2.** 25.030 **3.** 50.0250 **4.** 0.8

b) Change to Decimals: (Do not round off)

1. $\frac{7}{8}$ **2.** $\frac{3}{11}$ **3.** $\frac{4}{9}$ **4.** $\frac{3}{5}$ **5.** $\frac{4}{7}$

6. $\frac{5}{12}$ **7.** $\frac{5}{6}$ **8.** $\frac{2}{3}$ **9.** $\frac{7}{2}$ **10.** $\frac{2}{9}$

c) Round Off:

1. 2.864 to nearest hundredth

2. 35.9649 to nearest thousandth

3. 931.85 to nearest tenth

4. 2.091 to nearest tenth

5. 11.898 to nearest hundredth

6. 12.92 to nearest whole number

7. 11.74235 to nearest thousandth

8. $\frac{5}{9}$ to nearest thousandth

9. $\frac{7}{12}$ to nearest hundredth

ANSWERS

EXERCISE 1: Decimal Operations

a) **1)** $5\frac{4}{5}$ **2)** $27\frac{17}{50}$ **3)** $30\frac{1}{50}$ **4)** $\frac{59}{100}$ **5)** $3\frac{3}{40}$

b) **1)** 2.81 **2)** .05182 **3)** 16.992 **4)** 1758.534 **5)** 11.219

c) **1)** 2.442 **2)** .742 **3)** 80.887 **4)** .0502 **5)** 14.251

d) **1)** .0084 **2)** .09156 **3)** 99.5895 **4)** .018027 **5)** 72

6) 9432.5 **7)** .0094325

e) **1)** 310 **2)** 170 **3)** 20.5 **4)** .026 **5)** 750

6) .92356 **7)** 92356

EXERCISE 2: Conversion and Rounding Off

a) **1)** $2\frac{591}{1000}$ **2)** $25\frac{3}{100}$ **3)** $50\frac{1}{40}$ **4)** $\frac{4}{5}$

b) **1)** .875 **2)** $.\overline{27}$ **3)** $.\dot{4}$ **4)** .6 **5)** $.\overline{571428}$

6) $.41\dot{6}$ **7)** $.8\dot{3}$ **8)** $.\dot{6}$ **9)** 3.5 **10)** $.\dot{2}$

c) **1)** 2.86 **2)** 35.965 **3)** 931.9 **4)** 2.1 **5)** 11.90

6) 13 **7)** 11.742 **8)** .556 **9)** .58

EXPONENTS AND ROOTS

A. EXPONENTS

In math, many symbols have been developed to simplify certain types of number expressions. One of these symbols is the **"exponent"**.

Rule: An exponent indicates how many times a base number is used as a factor.

Example: $10^4 = 10 \times 10 \times 10 \times 10$ or $(10)(10)(10)(10)$
$3^5 = 3 \times 3 \times 3 \times 3 \times 3$
$x^2 = (x)(x)$

The exponent is written smaller and is placed above the base number (the number to be multiplied). The first example can be read "ten exponent four" or "ten to the power of four".

Second and third power have special names: second power is usually called "squared", and third power is usually called "cubed".

Example: 5^2 is "five squared"
5^3 is "five cubed"

A simple way to work out exponents is to write the base digit the same number of times as the value of the exponent, and put a multiplication sign between each digit.

Example: $4^5 = 4 \times 4 \times 4 \times 4 \times 4$ (the exponent is 5, so write 4 five times)
$= 1024$

PRACTICE A:

Write as an exponent or power (also called "*exponential notation*"):

1.) $5 \bullet 5 \bullet 5 \bullet 5 =$

2.) $2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 =$

3.) $10 \bullet 10 =$

4.) $6 \bullet 6 \bullet 6 =$

Evaluate the following:

5.) $2^4 =$

6.) $5^3 =$

7.) $3^4 =$

8.) $2^5 =$

9.) $10^2 =$

10.) $1^7 =$

11.) $\left(\frac{1}{4}\right)^2 =$

12.) $\frac{4^2}{5} =$

13.) $\left(\frac{4}{5}\right)^2 =$

14.) $10^3 + 4^2 =$

B. SQUARE ROOTS

Rule: Taking the square root of a number is the reverse of squaring the number.
The symbol for square root is $\sqrt{\ }$

Example: $\sqrt{9}$ (the square root of 9) = 3 because $(3)^2$ = 9
$\sqrt{144}$ (the square root of 144) = 12 because $(12)^2$ = 144

The square of a number can be positive (+) or negative (–). This is because two positive numbers multiplied together **or** two negative numbers multiplied together make a positive number. However, the $\sqrt{\ }$ symbol tells us we should write only the positive root.

Example: $\sqrt{4}$ = 2, but remember, $(+2)^2$ = 4 or $(-2)^2$ = 4
$\sqrt{25}$ = 5, but either $(+5)^2$ = 25 or $(-5)^2$ = 25

PRACTICE B:

Evaluate the following:

1.) $\sqrt{4}$ =
2.) $\sqrt{1}$ =
3.) $\sqrt{121}$ =
4.) $\sqrt{81}$ =
5.) $\sqrt{100}$ =
6.) $\sqrt{10000}$ =
7.) $\dfrac{\sqrt{36}}{49}$ =
8.) $\sqrt{\dfrac{36}{49}}$ =
9.) $\dfrac{\sqrt{81}}{100}$ =
10.) $\sqrt{\dfrac{81}{100}}$ =

ANSWERS

Practice A (exponents):

1.) 5^4
2.) 2^5
3.) 10^2
4.) 6^3
5.) 2 x 2 x 2 x 2 = 16
6.) 5 x 5 x 5 = 125
7.) 3 x 3 x 3 x 3 = 81
8.) 2 x 2 x 2 x 2 x 2 = 32
9.) 10 x 10 = 100
10.) 1 x 1 x 1 x 1 x 1 x 1 x 1 = 1
11.) $\dfrac{1}{4} \times \dfrac{1}{4} = \dfrac{1}{16}$
12.) $\dfrac{4x4}{5} = \dfrac{16}{5} = 3\dfrac{1}{5}$
13.) $\left(\dfrac{4}{5}\right) \times \left(\dfrac{4}{5}\right) = \dfrac{16}{25}$
14.) (10 x 10 x 10) + (4 x 4) = 1016

Practice B (square roots)

1.) 2
2.) 1
3.) 11
4.) 9
5.) 10
6.) 100
7.) $\dfrac{6}{49}$
8.) $\dfrac{6}{7}$
9.) $\dfrac{9}{100}$
10.) $\dfrac{9}{10}$

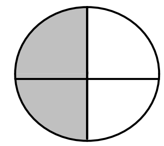
FRACTION REVIEW

A. INTRODUCTION

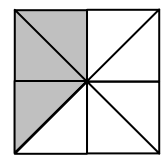
1. ***What is a fraction***?

Fraction = $\dfrac{\text{numerator}}{\text{denominator}}$

A fraction consists of a numerator (part) on top of a denominator (total) separated by a horizontal line.
For example, the fraction of the circle which is shaded is:



$$\frac{\textbf{2 (parts shaded)}}{\textbf{4 (total parts)}}$$

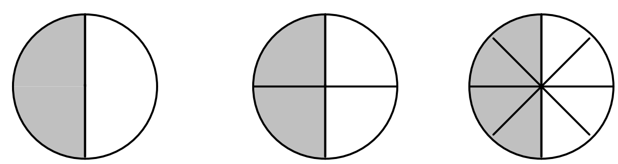


In the square on the right, the fraction shaded is $\frac{3}{8}$ and

the fraction unshaded is $\frac{5}{8}$

2. ***Equivalent Fractions – Multiplying***

The three circles on the right each have equal parts shaded, yet are represented by different but equal fractions. These fractions, because they are equal, are called <u>equivalent fractions</u>.



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{4}{8}$

Any fraction can be changed into an equivalent fraction
by multiplying **<u>both</u>** the numerator and denominator by the same number

$\frac{1\ \times 2}{2\ \times 2} = \frac{2}{4}$ or $\frac{1\ \times 4}{2\ \times 4} = \frac{4}{8}$ so $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

Similarly
$\frac{5\ \times 2}{9\ \times 2} = \frac{10}{18}$ or $\frac{5\ \times 3}{9\ \times 3} = \frac{15}{27}$ so $\frac{5}{9} = \frac{10}{18} = \frac{15}{27}$

You can see from the above examples that each fraction has an infinite number of fractions that are equivalent to it.

3. ***Equivalent Fractions – Dividing (Reducing)***

Equivalent fractions can also be created if both the numerator and denominator can be divided by the same number (a factor) evenly.
This process is called "reducing a fraction" by dividing a common factor (a number which divides into both the numerator and denominator evenly).

$$\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$$

$$\frac{27}{81} \div \frac{9}{9} = \frac{3}{9}$$

$$\frac{5}{30} \div \frac{5}{5} = \frac{1}{6}$$

$$\frac{6}{10} \div \frac{2}{2} = \frac{3}{5}$$

4. ***Simplifying a Fraction (Reducing to its Lowest Terms)***

It is usual to reduce a fraction until it can't be reduced any further.
A simplified fraction has no common factors which will divide into both numerator and denominator.
Notice that, since 27 and 81 have a common factor of 9, we find that $\frac{3}{9}$ is an equivalent fraction.
But this fraction has a factor of 3 common to both numerator and denominator.
So, we must reduce this fraction again. It is difficult to see, but if we had known that 27 was a factor (divides into both parts of the fraction evenly), we could have arrived at the answer in one step

e.g. $\frac{8}{24} \div \frac{8}{8} = \frac{1}{3}$ $\qquad$ $\frac{45}{60} \div \frac{15}{15} = \frac{3}{4}$

$$\frac{27}{81} \div \frac{9}{9} = \frac{3}{9}$$

$$\frac{3}{9} \div \frac{3}{3} = \frac{1}{3}$$

$$\frac{27}{81} \div \frac{27}{27} = \frac{1}{3}$$

5. ***EXERCISE 1: Introduction to Fractions***

a) Find the missing part of these equivalent fractions

1) $\frac{2}{3} = \frac{\ }{6}$ 2) $\frac{3}{4} = \frac{\ }{12}$ 3) $\frac{5}{8} = \frac{\ }{40}$

4) $\frac{1}{16} = \frac{\ }{32}$ 5) $\frac{2}{15} = \frac{\ }{45}$ 6.) $\frac{7}{9} = \frac{\ }{27}$

7) $\frac{7}{10} = \frac{\ }{100}$ 8) $\frac{3}{4} = \frac{\ }{44}$

x 2

Example: $\frac{3}{5} = \frac{\ }{10}$

x 2

Since 5 x 2 = 10, multiply the numerator by 2, also.

So, $\frac{3}{5} = \frac{6}{10}$

b) Find the missing part of these equivalent fractions.

1) $\frac{8}{16} = \frac{\ }{4}$ 2) $\frac{24}{27} = \frac{\ }{9}$

3) $\frac{6}{10} = \frac{\ }{5}$ 4) $\frac{25}{35} = \frac{\ }{7}$

5) $\frac{20}{30} = \frac{\ }{6}$ 6) $\frac{90}{100} = \frac{\ }{50}$

÷ 5

Example: $\frac{5}{10} = \frac{\ }{2}$

÷ 5

Since 10 ÷ 5 = 2 divide the numerator by 5, also.

So, $\frac{5}{10} = \frac{1}{2}$

c) Simplify the following fractions (reduce to lowest terms).

1) $\frac{9}{12}$ 2) $\frac{8}{12}$ 3) $\frac{6}{8}$ 4) $\frac{15}{20}$

5) $\frac{20}{25}$ 6) $\frac{14}{21}$ 7) $\frac{8}{16}$ 8) $\frac{24}{36}$

9) $\frac{66}{99}$ 10) $\frac{18}{30}$

B. TYPES OF FRACTIONS

1. ***Common Fractions***

A common fraction is one in which the numerator is less than the denominator
(or a fraction which is less than the number 1). A common fraction can also be called a proper fraction.

e.g. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{88}{93}$, $\frac{8}{15}$ are all common fractions.

2. ***Fractions that are Whole Numbers***

Some fractions, when reduced, are really whole numbers (1, 2, 3, 4… etc).
Whole numbers occur if the denominator divides into the numerator evenly.

e.g. $\frac{8}{4}$ *is the same as* $\frac{8}{4} \div \frac{4}{4} = \frac{2}{1}$ *or* 2

$\frac{30}{5}$ *is the same as* $\frac{30}{5} \div \frac{5}{5} = \frac{6}{1}$ *or* 6

So, the fraction $\frac{30}{5}$ is really the whole number 6.
Notice that a whole number can always be written as a fraction with a denominator of 1.

e.g. $10 = \frac{10}{1}$

3. ***Mixed Numbers***

A mixed number is a combination of a whole number and a common fraction.

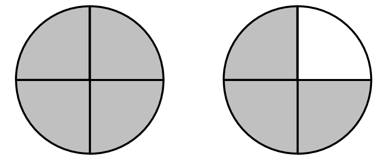
e.g. $2\frac{3}{5}$ (two and three-fifths)

$27\frac{2}{9}$ (twenty-seven and two-ninths)

$9\frac{3}{6} = 9\frac{1}{2}$ (always reduce fractions)

4. ***Improper Fractions***

An improper fraction is one in which the numerator is larger than the denominator.



$$1\frac{3}{4} = \frac{7}{4}$$

From the circles on the right, we see that $1\frac{3}{4}$ (mixed number) is the same as $\frac{7}{4}$ (improper fraction).

An improper fraction, like $\frac{7}{4}$, can be changed to a mixed number by dividing the denominator into the numerator and expressing the remainder (3) as the numerator.

$$\frac{7}{4} = 4\overline{)7}^{\,1} = 1\frac{3}{4}$$
$$\begin{array}{r} -4 \\ \hline 3 \end{array}$$

e.g. $\frac{16}{5} = 3\frac{1}{5}$ $\quad\frac{29}{8} = 3\frac{5}{8}$ $\quad\frac{14}{3} = 4\frac{2}{3}$

A mixed number can be changed to an improper fraction by changing the whole number to a fraction with the same denominator as the common fraction.

$$2\frac{3}{5} = \frac{10}{5} \text{ and } \frac{3}{5} \qquad 10\frac{1}{9} = \frac{90}{9} \text{ and } \frac{1}{9}$$
$$= \frac{13}{5} \qquad\qquad = \frac{91}{9}$$

A simple way to do this is to multiply the whole number by the denominator, and then add the numerator.

e.g.
$$4\frac{5}{9} = \frac{4 \text{ x } 9 + 5}{9} = \frac{36 + 5}{9} = \frac{41}{9}$$
$$10\frac{2}{7} = \frac{10 \text{ x } 7 + 2}{7} = \frac{70 + 2}{7} = \frac{72}{7}$$

5. ***Simplifying fractions***

All types of fractions must <u>always</u> be simplified (reduced to <u>lowest terms</u>).

e.g. $\frac{6}{9} = \frac{2}{3}$, $\quad 2\frac{5}{25} = 2\frac{1}{5}$, $\quad \frac{27}{18} = \frac{3}{2} = 1\frac{1}{2}$

Note that many fractions can not be reduced since they have no common factors.

e.g. $\frac{17}{21}, \frac{4}{9}, \frac{18}{37}$

6. ***EXERCISE 2 : Types of Fractions***

a) Which of the following are common fractions (C), whole numbers (W), mixed numbers (M) or improper fractions (I)?

1) $\frac{2}{3}$ 2) $3\frac{4}{5}$ 3) $\frac{7}{5}$ 4) $\frac{8}{8}$ 5) $\frac{24}{2}$

6) $5\frac{8}{19}$ 7) $2\frac{3}{3}$ 8) $\frac{25}{24}$ 9) $\frac{24}{25}$ 10) $\frac{12}{12}$

b) Change the following to mixed numbers:

1) $\frac{7}{5}$ 2) $\frac{18}{11}$ 3) $\frac{70}{61}$ 4) $\frac{12}{5}$ 5) $\frac{100}{99}$ 6) $\frac{25}{2}$

c) Change the following to improper fractions:

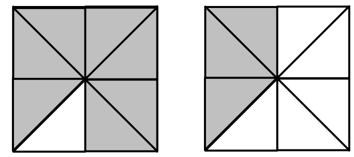
1) $2\frac{1}{5}$ 2) $6\frac{3}{8}$ 3) $8\frac{2}{3}$ 4) $11\frac{1}{5}$ 5) $9\frac{4}{5}$ 6) $4\frac{3}{4}$

d) Simplify the following fractions:

1) $\frac{28}{40}$ 2) $\frac{80}{10}$ 3) $2\frac{12}{18}$ 4) $5\frac{27}{54}$ 5) $\frac{25}{15}$ 6) $\frac{90}{12}$

C. COMPARING FRACTIONS

In the diagram on the right, it is easy to see that $\frac{7}{8}$ is larger than $\frac{3}{8}$ (since 7 is larger than 3).
However, it is not as easy to tell that $\frac{7}{8}$ is larger than $\frac{5}{6}$.



$\frac{7}{8}$ = $\frac{3}{8}$

In order to compare fractions, we must have the same (common) denominators. This process is called "<u>Finding the Least Common Denominator</u>" and is usually abbreviated as finding the <u>LCD</u> or LCM (lowest common multiple).

Which is larger: $\frac{7}{8}$ or $\frac{5}{6}$?

In order to compare these fractions, we must change both fractions to equivalent fractions with a common denominator.
To do this, <u>take the largest denominator (8) and examine multiples of it</u>, until the other denominator (6) divides into it.
Notice that, when we multiply 8 x 3, we get 24, which 6 divides into.
Now change the fractions to 24^{th} s.

8 x 1 = 8
(6 doesn't divide into 8)

8 x 2 = 16
(6 doesn't divide into 16)

8 x 3 = 24 LCD

$$\frac{7 \times 3}{8 \times 3} = \frac{21}{24} \qquad \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$

When we change these fractions to equivalent fractions with an LCD of 24, we can easily see that $\frac{7}{8}$ *is larger than* $\frac{5}{6}$ *since* $\frac{21}{24}$ *is greater than* $\frac{20}{24}$.

Which is larger: $\frac{4}{9}$ or $\frac{5}{12}$?

Examine multiples of the larger denominator (12) until the smaller denominator divides into it. This tells us that the LCD is 36. Now, we change each fraction to equivalent fractions with the LCD of 36.

12 x 1 = 12
12 x 2 = 24
12 x 3 = 36 (LCD)
LCD is 36.

$$\frac{4}{9} \underset{\times 4}{\overset{\times 4}{=}} \frac{16}{36} \qquad \frac{5}{12} \underset{\times 3}{\overset{\times 3}{=}} \frac{15}{36}$$

So, $\frac{4}{9}$ *is larger than* $\frac{5}{12}$

Which is larger: $\frac{4}{5}$ or $\frac{13}{15}$ or $\frac{11}{12}$?

Find the LCD by examining multiples of 15. Notice that, when we multiply 15 x 4, we find that 60 is the number that <u>all</u> denominators divide into.

15 x 1 = 15
15 x 2 = 30
15 x 3 = 45
15 x 4 = 60 (LCD)

$$\frac{4}{5} \underset{\times 12}{\overset{\times 12}{=}} \frac{48}{60} \qquad \frac{13}{15} \underset{\times 4}{\overset{\times 4}{=}} \frac{52}{60} \qquad \frac{11}{12} \underset{\times 5}{\overset{\times 5}{=}} \frac{55}{60}$$

So, $\frac{11}{12}$ *is the largest fraction.*

Which is larger: $\frac{7}{9}$ or $\frac{13}{18}$?

Notice that one denominator (9) divides into the other denominator (18). This means that the LCD = 18 and we only have to change one fraction $\left(\frac{7}{9}\right)$ to an equivalent fraction.

$$\frac{7}{9} \overset{\times 2}{\underset{\times 2}{=}} \frac{14}{18}$$

So, $\frac{7}{9}$ *is larger than* $\frac{13}{18}$

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1. ***<u>EXERCISE 3: Comparing Fractions</u>***

Which is the largest fraction? (find LCD first)

1) $\frac{7}{13}$ *or* $\frac{6}{13}$

2) $1\frac{9}{10}$ *or* $\frac{18}{10}$

3) $\frac{4}{5}$ *or* $\frac{9}{10}$

4) $\frac{3}{13}$ *or* $\frac{5}{12}$

5) $\frac{5}{8}$ *or* $\frac{4}{7}$

6) $\frac{1}{2}$ *or* $\frac{6}{11}$ *or* $\frac{7}{12}$

7) $\frac{2}{3}$ *or* $\frac{11}{15}$

8) $\frac{4}{9}$ *or* $\frac{5}{12}$ *or* $\frac{3}{8}$

9) $\frac{1}{4}$ *or* $\frac{3}{16}$

D. ADDING FRACTIONS

There are four main operations that we can do with numbers: addition (+), subtraction (–), multiplication (x), and division (÷).
In order to add or subtract, fractions must have common denominators.
This is not required for multiplication or division.

1. *Adding with Common Denominators*

To add fractions, if the denominators are the same, we simply add the numerators and keep the same denominators.

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

e.g. Add $\frac{1}{12}$ and $\frac{5}{12}$

Since the denominators are common, simply add the numerators. Notice that we must reduce the answer, if possible.

$$\frac{1}{12} + \frac{5}{12} = \frac{6}{12} = \frac{1}{2}$$

2. *Adding When One Denominator is a Multiple of the Other*

Add $\frac{2}{9}$ and $\frac{5}{27}$

$$\frac{2 \times 3}{9 \times 3} = \frac{6}{27}$$

Notice that the denominators are not common. Also notice that 27 is a multiple of 9 (since 9 x 3 = 27). This means that the LCD = 27 (see the last example in "Comparing Fractions").

$$\frac{2}{9} + \frac{5}{27} = \frac{6}{27} + \frac{5}{27} = \frac{11}{27}$$

3. ***<u>Adding Any Fraction</u>***

Add $\frac{7}{12}$ and $\frac{13}{15}$

$$\frac{7}{12} + \frac{13}{15} = \frac{35}{60} + \frac{52}{60} = \frac{87}{60} = 1\frac{9}{20}$$

We must find a common denominator by examining multiples of the largest denominator. We find that the LCD = 60.

Add $1\frac{5}{6}$ and $2\frac{3}{8}$

When adding mixed numbers, add the whole numbers and the fractions separately. Find common denominators and add.

$$\begin{array}{rcl} 1\frac{5}{6} & = & 1\frac{20}{24} \\ +\quad 2\frac{3}{8} & = & 2\frac{9}{24} \\ \hline \end{array}$$

total equals $3\frac{29}{24}$

If an improper fraction occurs in the answer, change it to a common fraction by doing the following.

$$3\frac{29}{24} = 3 + 1\frac{5}{24} = 4\frac{5}{24}$$

4. ***<u>The Language of Addition</u>***

$\frac{1}{2} + \frac{2}{3}$ *CAN BE WORDED*

$\frac{1}{2}$ *<u>plus</u>* $\frac{2}{3}$ $\qquad$ $\frac{1}{2}$ *<u>and</u>* $\frac{2}{3}$ $\qquad$ *<u>total</u> of* $\frac{1}{2}$ *and* $\frac{2}{3}$ $\qquad$ *<u>sum</u> of* $\frac{1}{2}$ *and* $\frac{2}{3}$

<u>addition</u> of $\frac{1}{2}$ *and* $\frac{2}{3}$ $\qquad$ $\frac{1}{2}$ *<u>combined with</u>* $\frac{2}{3}$ $\qquad$ $\frac{1}{2}$ *<u>more than</u>* (or greater than) $\frac{2}{3}$

<u>Note:</u> All of these can be worded with the fractions in reverse order:

e.g. $\frac{2}{3}$ *plus* $\frac{1}{2}$ *is the same as* $\frac{1}{2}$ *plus* $\frac{2}{3}$

5. ***EXERCISE 4: Adding Fractions***

a) Add the following:

1) $\frac{1}{5} + \frac{2}{5}$ 2) $\frac{4}{5} + \frac{3}{5}$

3) $\frac{4}{9} + \frac{2}{9}$ 4) $\frac{3}{4} + \frac{3}{4} + \frac{3}{4}$

b) Find the sum of:

1) $\frac{2}{3} + \frac{1}{9}$ 2) $\frac{1}{2} + \frac{3}{8}$

3) $\frac{1}{4} + \frac{5}{16}$ 4) $\frac{2}{3} + \frac{4}{15}$

c) Add the following:

1) $3\frac{1}{2} + 4\frac{1}{4}$ 2) $9\frac{2}{3} + 3\frac{1}{6}$

3) $8\frac{1}{2} + 4\frac{4}{5}$ 4) $2\frac{3}{4} + 6\frac{1}{2}$

5) $4\frac{1}{3} + 6\frac{5}{6}$ 6) $6\frac{1}{3} + 8\frac{3}{4}$

7) $7\frac{2}{3} + \frac{4}{5}$ 8) $8\frac{2}{3} + 6\frac{1}{4} + 1\frac{3}{8}$

d) Evaluate the following:

1) $\frac{2}{3}$ *and* $\frac{3}{7}$ 2) *total of* $\frac{5}{6}$ *and* $\frac{3}{8}$

3) $\frac{1}{2}$ *plus* $\frac{1}{5}$ 4) $\frac{3}{2}$ *greater than* $\frac{5}{7}$

5) $\frac{5}{12}$ *combined with* $\frac{3}{8}$ 6) *sum of* $\frac{1}{6}$ *and* $\frac{3}{14}$

E. SUBTRACTING FRACTIONS

1. ***Common Fractions***

As in addition, we must have common denominators in order to subtract. Find the LCD; change the fractions to equivalent fraction with the LCD as the denominator. Then subtract the numerators, but keep the same denominator.

$$\frac{5}{8} - \frac{3}{8} = \frac{2}{8} \text{ or } \frac{1}{4}$$

$$\frac{2}{3} - \frac{3}{8} = \frac{16}{24} - \frac{9}{24} = \frac{7}{24}$$

2. ***Mixed Numbers***

When subtracting whole numbers, subtract the whole numbers, and then subtract the fractions separately.

$$3\frac{5}{9} - 1\frac{3}{9} = 2\frac{2}{9}$$

However, if the common fraction we are subtracting is smaller than the other common fraction, we must borrow the number "1" from the large whole number.

i.e. $4\frac{2}{7} = 3 + \frac{7}{7} + \frac{2}{7}$, or $3\frac{9}{7}$

$$4\frac{2}{7} - 2\frac{5}{7} = 3\frac{9}{7} - 2\frac{5}{7} = 1\frac{4}{7}$$

To subtract $1\frac{3}{4}$ ***from*** $6\frac{2}{3}$, first change the common fractions to equivalent fractions with the LCD. Since $\frac{8}{12}$ is smaller than $\frac{9}{12}$, borrow from 6.

$$6\frac{8}{12} = 5\frac{12}{12} + \frac{8}{12} = 5\frac{20}{12}$$

$$6\frac{2}{3} - 1\frac{3}{4} = 6\frac{8}{12} - 1\frac{9}{12} = 5\frac{20}{12} - 1\frac{9}{12} = 4\frac{11}{12}$$

3. ***The Language of Subtraction***

$\frac{5}{6} - \frac{2}{3}$ *CAN BE WORDED*

$\frac{5}{6}$ *minus* $\frac{2}{3}$ (*NOT* $\frac{2}{3}$ *minus*)

$\frac{2}{3}$ *subtracted from* $\frac{5}{6}$

$\frac{2}{3}$ *from* $\frac{5}{6}$

$\frac{2}{3}$ *less than* $\frac{5}{6}$

$\frac{5}{6}$ *decreased by* or *lowered by* $\frac{2}{3}$

the difference of $\frac{5}{6}$ *and* $\frac{2}{3}$

NOTE: Unlike addition, we can not reword the above with the fractions in reverse order:

i.e. $\frac{1}{2} - \frac{2}{3}$ **is NOT the same as** $\frac{2}{3} - \frac{1}{2}$

==

4. ***EXERCISE 5: Subtracting Fractions***

a) Subtract the following:

1) $\frac{9}{12} - \frac{1}{8}$ 2) $\frac{14}{15} - \frac{1}{6}$ 3) $\frac{5}{6} - \frac{3}{8}$

4) $\frac{7}{9} - \frac{2}{3}$ 5) $9\frac{2}{3} - 6\frac{1}{6}$ 6) $4\frac{1}{2} - 1\frac{1}{4}$

7) $\frac{3}{4} - \frac{5}{8}$ 8) $\frac{11}{12} - \frac{2}{3}$

b) Subtract the following:

1) $6\frac{1}{3} - 2\frac{2}{3}$ 2) $13\frac{1}{4} - 5\frac{3}{4}$ 3) $5\frac{5}{7} - 4\frac{6}{7}$

4) $4\frac{3}{4} - 1\frac{11}{12}$ 5) $16\frac{2}{3} - 5\frac{3}{4}$ 6) $9\frac{1}{6} - 4\frac{3}{8}$

c) Find the following:

1) What is $\frac{5}{8}$ minus $\frac{3}{16}$? 2) $\frac{2}{7}$ decreased by $\frac{1}{21}$ is what?

3) What is $\frac{4}{9}$ less than $\frac{7}{9}$? 4) What is $\frac{1}{6}$ from $\frac{9}{24}$?

F. MULTIPLYING FRACTIONS

1. *Common Fractions*

When multiplying fractions, a common denominator is not needed. Simply multiply the numerators and multiply the denominators separately.

$$\frac{2}{3} \text{ x } \frac{5}{9} = \frac{2\times5}{3\times9} = \frac{10}{27}$$

Sometimes, we can reduce the fractions before multiplying.

$$\frac{3}{5} \text{ x } \frac{5}{7} = \frac{3\times\cancel{5}^{1}}{\cancel{5}_{1}\times7} = \frac{3}{7}$$

Any common factor in either numerator can cancel with the same factor in the denominator. Multiply after cancelling (reducing).

$$\frac{4}{9} \text{ x } \frac{3}{8} = \frac{\cancel{4}^{1}\times\cancel{3}^{1}}{\cancel{9}_{3}\times\cancel{8}_{2}} = \frac{1}{6}$$

Note that any whole number (16) has the number "1" understood in its denominator.

$$\frac{5}{8} \text{ x } 16 = \frac{5\times\cancel{16}^{2}}{\cancel{8}_{1}\times1} = 10$$

If more than two fractions are multiplied, the same principles apply.

$$\frac{2}{9} \text{ x } \frac{3}{20} \text{ x } \frac{1}{4} = \frac{2\times3\times1}{9\times20\times4} = \frac{1}{120}$$

2. *Mixed Numbers*

Mixed numbers must be changed to improper fractions before multiplying. Remember that a mixed number (like $2\frac{3}{4}$) can be changed to an improper fraction by multiplying the whole number (2) by the denominator (4) and then adding the numerator.

See page 6 for instructions on changing a mixed number to an improper fraction.

$$2\frac{3}{4} \text{ x } 4\frac{8}{11} = \frac{11}{4} \text{ x } \frac{52}{11}$$
$$= 13$$

$$5\frac{4}{9} \text{ x } 2\frac{3}{7} = \frac{49}{9} \text{ x } \frac{17}{7}$$
$$= \frac{119}{9}$$
$$\text{or } 13\frac{2}{9}$$

3. ***The Language of Multiplication***

$\frac{1}{2}$ x $\frac{2}{3}$ *CAN BE WORDED*

$\frac{1}{2}$ *multiplied by* $\frac{2}{3}$

$\frac{1}{2}$ *by* $\frac{2}{3}$

$\frac{1}{2}$ *of* $\frac{2}{3}$

the product of $\frac{1}{2}$ *and* $\frac{2}{3}$

NOTE: When multiplying, it doesn't matter which fraction is first.

i.e. $\frac{1}{2} \times \frac{2}{3}$ **is the same as** $\frac{2}{3} \times \frac{1}{2}$

==

4. ***EXERCISE 6: Multiplying Fractions***

a) Multiply (cancel first, when possible):

1) $\frac{2}{3} \times \frac{3}{4}$

2) $\frac{8}{9} \times \frac{12}{16}$

3) $\frac{5}{3} \times \frac{9}{15}$

4) $\frac{2}{5} \times \frac{15}{21}$

5) $\frac{5}{8} \times \frac{48}{125}$

6) $\frac{3}{4} \times \frac{16}{27} \times \frac{9}{16}$

7) $\frac{5}{3} \times \frac{7}{4} \times 90 \times 20 \times \frac{16}{14}$

8) $2\frac{4}{5} \times 3\frac{4}{7}$

9) $8\frac{2}{3} \times \frac{15}{21}$

10) $\frac{18}{19} \times \frac{57}{4} \times \frac{6}{32} \times \frac{8}{9} \times \frac{2}{3}$

11) $1\frac{12}{25} \times 2\frac{1}{37}$

12) $2\frac{5}{12} \times 6$

b) Find the following:

1) What is $\frac{2}{3}$ of 45?

2) $\frac{41}{7}$ by 14 is what number?

3) What is $\frac{5}{9}$ times $\frac{27}{10}$?

4) $\frac{4}{5}$ of $2\frac{1}{7}$ is what number?

5) $\frac{1}{2}$ of $\frac{62}{63}$ is what number?

G. DIVIDING FRACTIONS

To divide fractions, we invert (take the reciprocal of) the fraction that we are dividing by, then cancel (reduce), and then multiply.
Taking the reciprocal of a fraction involves "flipping" the fraction so that the numerator and denominator switch places.

$\frac{2}{3} \xrightarrow{\textit{reciprocal}} \frac{3}{2}$

$\frac{8}{19} \xrightarrow{\textit{reciprocal}} \frac{19}{8}$

Note that a whole number is really a fraction (e.g. $4 = \frac{4}{1}$).

$4 \xrightarrow{\textit{reciprocal}} \frac{4}{1}$

1. *Common Fractions*

Simply invert (take the reciprocal of) the fractions that we are dividing by ($\frac{8}{9}$). Then cancel and multiply.

Note: you can only cancel **after** the division is changed to a multiplication.

$$\frac{5}{7} \div \frac{8}{9} = \frac{5}{7} \times \frac{9}{8} = \frac{45}{56}$$

$$\frac{9}{16} \div \frac{3}{32} = \frac{9}{16} \times \frac{32}{3} = 6$$

2. *Mixed Numbers*

As in multiplication, mixed numbers must be changed to improper fractions.

$$1\frac{2}{3} \div 2\frac{1}{7} = \frac{5}{3} \div \frac{15}{7}$$
$$= \frac{5}{3} \times \frac{7}{15}$$
$$= \frac{7}{9}$$

3. ***The Language of Division***

$\frac{1}{2} \div \frac{2}{3}$ *CAN BE WORDED*

$\frac{1}{2}$ *divided by* $\frac{2}{3}$ $\quad$ $\frac{2}{3}$ *into* $\frac{1}{2}$ $\quad$ *divide* $\frac{1}{2}$ *by* $\frac{2}{3}$

NOTE: In multiplication, the order of the fractions was not important.

i.e. $\frac{1}{2} \text{ x } \frac{2}{3}$ **is the same as** $\frac{2}{3} \text{ x } \frac{1}{2}$

In division, this is not the case. The order of the fractions is important.
Consider the following:

$$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

$$\text{but } \frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \times \frac{2}{1} = \frac{4}{3} = 1\frac{1}{3}$$

===

4. ***EXERCISE 7: Dividing Fractions***

a) Divide:

1) $\frac{3}{5} \div \frac{9}{15}$ $\quad$ 2) $\frac{3}{7} \div \frac{12}{5}$ $\quad$ 3) $6 \div \frac{2}{3}$

4) $\frac{7}{12} \div 14$ $\quad$ 5) $\frac{13}{15} \div \frac{39}{40}$ $\quad$ 6) $3\frac{3}{10} \div \frac{11}{15}$

7) $\frac{27}{30} \div 8\frac{1}{3}$ $\quad$ 8) $1\frac{7}{9} \div 1\frac{2}{3}$ $\quad$ 9) $4\frac{2}{3} \div 3\frac{1}{2}$

10) $2\frac{3}{11} \div 3\frac{9}{22}$

b) Find the following:

1) $\frac{4}{5}$ divided by $\frac{2}{5}$ $\quad$ 2) $\frac{2}{5}$ divided by $\frac{4}{5}$ $\quad$ 3) $\frac{9}{4}$ into $\frac{12}{25}$

4) Divide $1\frac{2}{3}$ by $\frac{2}{3}$

5) $\frac{4}{9}$ into $1\frac{4}{5}$

6) Divide $\frac{1}{4}$ by $\frac{2}{5}$

FRACTION REVIEW:

Decide which operation (+, -, x, $\div$) by the wording in the question. Then find the answer.

1) What is $\frac{3}{5}$ of 40?

2) How much is $4\frac{2}{3}$ from $6\frac{1}{5}$?

3) How much is $\frac{1}{2}$ from $\frac{2}{3}$?

4) How much is $\frac{3}{7}$ of 21?

5) $\frac{4}{5}$ and $\frac{2}{3}$ equals….?

6) $\frac{2}{3}$ divided by 14 is what number?

7) What is $\frac{9}{10}$ of 300?

8) What is $\frac{2}{5}$ into 12?

9) $\frac{3}{4}$ from $\frac{8}{9}$ equals….?

10) What is $1\frac{2}{3}$ by $3\frac{3}{8?}$

11) How much is $\frac{5}{7}$ of $\frac{21}{50}$?

12) Find the total of $\frac{2}{3}$ and $\frac{1}{6}$ and $\frac{4}{9}$?

13) $\frac{3}{4}$ of $\frac{2}{3}$ equals….?

14) What is $\frac{2}{5}$ greater than $4\frac{3}{5}$?

ANSWER KEY – FRACTION REVIEW

EXERCISE 1: Introduction to Fractions (Page 3)

a) 1) $\frac{4}{6}$ 2) $\frac{9}{12}$ 3) $\frac{25}{40}$ 4) $\frac{2}{32}$ 5) $\frac{6}{45}$ 6) $\frac{21}{27}$ 7) $\frac{70}{100}$ 8) $\frac{33}{44}$

b) 1) $\frac{2}{4}$ 2) $\frac{8}{9}$ 3) $\frac{3}{5}$ 4) $\frac{5}{7}$ 5) $\frac{4}{6}$ 6) $\frac{45}{50}$

c) 1) $\frac{3}{4}$ 2) $\frac{2}{3}$ 3) $\frac{3}{4}$ 4) $\frac{3}{4}$ 5) $\frac{4}{5}$ 6) $\frac{2}{3}$ 7) $\frac{1}{2}$ 8) $\frac{2}{3}$ 9) $\frac{2}{3}$ 10) $\frac{3}{5}$

EXERCISE 2: Types of Fractions (Page 6)

a) 1) C 2) M 3) I 4) W 5) W 6) M 7) W 8) I 9) C 10) W

b) 1) $1\frac{2}{5}$ 2) $1\frac{7}{11}$ 3) $1\frac{9}{61}$ 4) $2\frac{2}{5}$ 5) $1\frac{1}{99}$ 6) $12\frac{1}{2}$

c) 1) $\frac{11}{5}$ 2) $\frac{51}{8}$ 3) $\frac{26}{3}$ 4) $\frac{56}{5}$ 5) $\frac{49}{5}$ 6) $\frac{19}{4}$

d) 1) $\frac{7}{10}$ 2) 8 3) $2\frac{2}{3}$ 4) $5\frac{1}{2}$ 5) $\frac{5}{3}$ or $1\frac{2}{3}$ 6) $\frac{15}{2}$ or $7\frac{1}{2}$

EXERCISE 3: Comparing Fractions (Page 9)

1) $\frac{7}{13}$ 2) $1\frac{9}{10}$ 3) $\frac{9}{10}$ 4) $\frac{5}{12}$ 5) $\frac{5}{8}$ 6) $\frac{7}{12}$ 7) $\frac{11}{15}$ 8) $\frac{4}{9}$ 9) $\frac{1}{4}$

EXERCISE 4: Adding Fractions (Page 12)

a) 1) $\frac{3}{5}$ 2) $\frac{7}{5}$ or $1\frac{2}{5}$ 3) $\frac{2}{3}$ 4) $\frac{9}{4}$ or $2\frac{1}{4}$

b) 1) $\frac{7}{9}$ 2) $\frac{7}{8}$ 3) $\frac{9}{16}$ 4) $\frac{14}{15}$

c) 1) $7\frac{3}{4}$ 2) $12\frac{5}{6}$ 3) $13\frac{3}{10}$ 4) $9\frac{1}{4}$ 5) $11\frac{1}{6}$ 6) $15\frac{1}{12}$ 7) $8\frac{7}{15}$ 8) $16\frac{7}{24}$

d) 1) $1\frac{2}{21}$ 2) $1\frac{5}{24}$ 3) $\frac{7}{10}$ 4) $2\frac{3}{14}$ 5) $\frac{19}{24}$ 6) $\frac{8}{21}$

EXERCISE 5: Subtracting Fractions (Page 14)

a) 1) $\frac{5}{8}$ 2) $\frac{23}{30}$ 3) $\frac{11}{24}$ 4) $\frac{1}{9}$ 5) $3\frac{1}{2}$ 6) $3\frac{1}{4}$ 7) $\frac{1}{8}$ 8) $\frac{1}{4}$

b) 1) $3\frac{2}{3}$ 2) $7\frac{1}{2}$ 3) $\frac{6}{7}$ 4) $2\frac{5}{6}$ 5) $10\frac{11}{12}$ 6) $4\frac{19}{24}$

c) 1) $\frac{7}{16}$ 2) $\frac{5}{21}$ 3) $\frac{1}{3}$ 4) $\frac{5}{24}$

EXERCISE 6: Multiplying Fractions (Page 16)

a) 1) $\frac{1}{2}$ 2) $\frac{2}{3}$ 3) 1 4) $\frac{2}{7}$ 5) $\frac{6}{25}$ 6) $\frac{1}{4}$ 7) 6000

8) 10 9) $\frac{130}{21}$ or $6\frac{4}{21}$ 10) $1\frac{1}{2}$ 11) 3 12) $\frac{29}{2}$ or $14\frac{1}{2}$

b) 1) 30 2) 82 3) $\frac{3}{2}$ or $1\frac{1}{2}$ 4) $\frac{12}{7}$ or $1\frac{5}{7}$ 5) $\frac{31}{63}$

EXERCISE 7: Dividing Fractions (Page 18)

a) 1) 1 2) $\frac{5}{28}$ 3) 9 4) $\frac{1}{24}$ 5) $\frac{8}{9}$ 6) $4\frac{1}{2}$ 7) $\frac{27}{250}$

8) $1\frac{1}{15}$ 9) $1\frac{1}{3}$ 10) $\frac{2}{3}$

b) 1) 2 2) $\frac{1}{2}$ 3) $\frac{16}{75}$ 4) $2\frac{1}{2}$ 5) $4\frac{1}{20}$ 6) $\frac{5}{8}$

Fraction Review (Page 19)

a) 1) 24 2) $1\frac{8}{15}$ 3) $\frac{1}{6}$ 4) 9 5) $1\frac{7}{15}$ 6) $\frac{1}{21}$ 7) 270

8) 30 9) $\frac{5}{36}$ 10) $5\frac{5}{8}$ 11) $\frac{3}{10}$ 12) $1\frac{5}{18}$ 13) $\frac{1}{2}$ 14) 5

PERCENT REVIEW

The use of percentage is another way of expressing numbers (usually fractions) in such a way as to make comparisons between them more obvious. For instance, if you get 28 out of 40 in test A and 37 out of 50 in Test B, it may not be clear whether you have improved or not. The use of percentage will allow this comparison, because a percent is part of 100. (i.e. a percent is a fraction with a denominator of 100).

A. CHANGING % TO FRACTIONS / DECIMALS

A percent means a part of 100. For example, if you get 95% on a test, your mark was 95 out of 100. A percent can be changed to a fraction or decimal by simply dividing the percentage number by 100.

1. Changing % to Fractions

Divide by 100.
(i.e. put the % number over 100 and reduce if necessary)

$$93\% = \frac{93}{100}$$

$$50\% = \frac{50}{100} = \frac{1}{2} \qquad 24\% = \frac{24}{100} = \frac{6}{25}$$

If a decimal appears in the fraction, multiply the fraction by 10, 100, 1000 etc. to produce an equivalent fraction without decimals.

$$26.3\% = \frac{26.3}{100} \times \frac{10}{10} = \frac{263}{1000}$$

$$5.55\% = \frac{5.55}{100} \times \frac{100}{100} = \frac{555}{10000} = \frac{111}{2000}$$

2. Changing % to Decimals

Simply divide by 100. (i.e. move the decimal point 2 places to the left)

50% = .50 or .5

9.23% = .0923

4% = .04

148% = 1.48

B. CHANGING FRACTIONS / DECIMALS TO PERCENTS

1. Changing Fractions to Percent

If you get 17 out of 20 on a test, it is convenient to change this mark to a percentage. This means changing $\frac{17}{20}$ to an equivalent fraction with 100 as denominator (i.e. $\frac{17}{20} = \frac{?}{100}$).

To change fractions to %, simply multiply the fraction by 100%.

$\frac{17}{20} = \frac{17}{20}$ x 100% = $\frac{1700}{20}$ = 85%

$\frac{1}{2} = \frac{1}{2}$ x 100% = 50%

$\frac{2}{3} = \frac{2}{3}$ x 100% = $66.\dot{6}$%

$\frac{19}{40} = \frac{19}{40}$ x 100% = $47\frac{1}{2}$% or 47.5%

Note: The mathematical wording for changing a fraction ($\frac{17}{20}$) to a percent would normally be:

17 is what % (out) of 20?
or
What % is 17 (out) of 20?

The word “out” is usually not included.

e.g. 19 is what % of 75?

$\frac{19}{75} = \frac{19}{75}$ x 100% = $25\frac{1}{3}$%

What % is 7 of 5?

$\frac{7}{5} = \frac{7}{5}$ x 100% = 140%

2. Changing Decimals to Percents

To change decimals to percents, simply multiply by 100% (i.e. move the decimal point 2 places to the right.)

.29	=	.29	x	100%	=	29%
.156	=	.156	x	100%	=	15.6%
1.3	=	1.3	x	100%	=	130%

C. USING PERCENTS

When percents are used in calculations, they are first converted to either fractions or decimals. Usually it is more convenient to change % to decimals.

1. Multiplying With Percents

If a test mark was 50% and it was out of 40 total marks, what was the test score?

$$50\% = \frac{\text{test score?}}{40\ (\text{total marks})}$$

$$50\% \times 40 = .5 \times 40 = 20$$

$$\text{So}\quad 50\% = \frac{20\text{ marks}}{40}$$

50% (out) of 40 is what number?

What number is 50% (out) of 40?

To find the test score, or the part, we multiply the % by the total.

e.g. 85% of 25 is what number?	85% x 25 =	.85 x 25 = 21.25
What number is 30% of 45.37?	30% x 45.37 =	.3 x 45.37 = 13.611

2. Dividing with Percents

If a test mark was 50% and you received a score of 20 marks, what was the test out of?

$$50\% = \frac{20\text{ marks}}{\text{total?}}$$

$$20 \div 50\% = 20 \div .5 = 40$$

$$\text{or } 20 \div \frac{1}{2} = 20 \times \frac{2}{1} = 40$$

$$\text{So, } 50\% = \frac{20}{40}\ \text{total marks}$$

50% (out) of what number is 20?

20 is 50% (out) of what number?

To find the total marks, we divide by the %.

e.g. 40% of what number is 25?	25 ÷ 40% = 25 ÷ .40 =	62.5
18 is 75% of what number?	18 ÷ 75% = 18 ÷ .75 =	24

D. SUMMARY AND EXERCISE

1. Three types of Percent Problems

In summary, there are three things that we can do with percent. We will use the example on the right side of the page to summarize.

$$50\% = \frac{20 \text{ (part)}}{40 \text{ total}}$$

1. Finding % **or** what % of 40 is 20? $\frac{20}{40} = \frac{20}{40} \times 100\% = 50\%$

2. Finding the Part **or** 50% of 40 is what number? $50\% \times 40 = .5 \times 40 = 20$

3. Finding the total **or** 50% of what number is 20? $20 \div 50\% = 20 \div .5 = 40$

===

2. EXERCISE: PERCENT PROBLEMS

1. **Change to Fractions**

a) 97% b) 82% c) 150% d) 45.3%

e) 9.25% f) 40% g) $5\frac{1}{2}\%$

2. **Change to Decimals**

a) 42% b) 9.37% c) 2% d) 243.9%

e) 0.95%

3. **Change to %**

a) $\frac{19}{20}$ b) $\frac{2}{3}$ c) $\frac{18}{75}$ d) $\frac{1}{12}$

e) $\frac{5}{9}$ f) $\frac{38}{40}$ g) 0.865 h) 2.37

i) .0092 j) $\frac{7}{4}$

4. **Finding %**

a) What % of 72 is 18?

b) 16 is what % of 80?

c) What % of 30 is 18.5?

5. **Finding the Part**

a) 40% of 18 is what number?

b) What number is 16.5% of 30.2?

c) 65% of 15 is what?

6. **Finding the Total**

a) 40% of what number is 12?

b) 18 is 55% of what number?

c) 120 is 150% of what number?

7. **Percent Problems Combined**

a) What % of 25 is 5?

b) 70% of 15 is what number?

c) 85 is 20% of what number?

d) 90 is what % of 55?

e) 30% of what number is 80?

f) What number is 42% of 50?

ANSWERS

1. a) $\frac{97}{100}$ b) $\frac{41}{50}$ c) $1\frac{1}{2}$ d) $\frac{453}{1000}$ e) $\frac{37}{400}$ f) $\frac{2}{5}$ g) $\frac{11}{200}$

2. a) .42 b) .0937 c) .02 d) 2.439 e) .0095

3. a) 95% b) $66.\dot{6}\%$ or $66\frac{2}{3}\%$ c) 24% d) $8.\dot{3}\%$ e) $55.\dot{5}\%$ f) 95%
   g) 86.5% h) 237% i) .92% j) 175%

4. a) 25% b) 20% c) $61.\dot{6}\%$

5. a) 7.2 b) 4.983 c) 9.75

6. a) 30 b) $32.\overline{72}$ c) 80

7. a) 20% b) 10.5 c) 425 d) $163.\overline{63}\%$ e) $266.\dot{6}$ f) 21

Section 2

ALGEBRA REVIEW

PART 1 - SIGNED NUMBERS

In the REAL NUMBER SYSTEM, numbers can be either **positive or negative.**
Positive 5 can be written as: +5, or (+5), or just 5.
Negative 5 can be written as: –5, or (–5).

Part 2 - ADDING and SUBTRACTING SIGNED NUMBERS

1. When adding numbers of the same sign, simply put the numbers together and carry the sign.

(+ 4) + (+ 2) = +6	(–4) + (–2) = –6
– 4 – 2 = –6	(–9) – 5 = –14
	Note: This reads –9 **combined with** –5

2. When adding numbers of different signs, take the difference between the two numbers; carry the sign of the number with the largest absolute value.

   For example: –9 + 2. The difference is 7. Take the sign of the 9. Answer = (–7)

   1) 4 – 2 = 2
   2) –2 + 4 = 2
   3) –7 + 3 = –4
   4) 12 + (–3) = 9

   NOTE: These two examples are the same, just switched around!

EXERCISE ON PART 2: ADDING AND SUBTRACTING SIGNED NUMBERS

1) –9 + 2	2) –3 + (-5)	3) –5/8 + 1/4
4) –6 + (-8)	5) 0 – 10	6) –8 – 3
7) 18 – 63	8) –49 + (-4)	9) –2/3 - 3/4
10) (–5) + (12) – 7	11) 3.5 – 2.2 + (– 4.0)	12) 15 + (–2) – 7 + 14 – 5 + (–12)

Part 3: MULTIPLYING AND DIVIDING WITH SIGNED NUMBERS

1. The rules for multiplying and dividing two numbers at a time are the same. They are as follows:

   I. When the signs are the same the answer is positive.
   II. When the signs are different the answer is negative.

1) $-6 \bullet (-5) = 30$
2) $6 \bullet (-5) = -30$
3) $(-4)(3) = -12$
4) $-\dfrac{14}{21} = -\dfrac{2}{3}$
5) $-36 \div -6 = 6$
6) $7 \div (-2) = -3.5$
7) $\dfrac{-25}{-5} = 5$
8) $-\dfrac{3}{4} = -0.75$

REMOVING BRACKETS

When doing operations with more than two signed numbers, remove brackets first before doing any operations. Then, simply take two numbers at a time.

To remove brackets: if a positive sign is outside the brackets simply remove the brackets.

e.g. +(7) becomes 7 *and* +(–7) becomes –7

if a negative sign is outside the brackets, **change the sign of the number in the brackets.**

e.g. – (+5) becomes –5 *and* – (–5) becomes 5

1) $+(-5) - (-7) = -5 + 7$
   $= \mathbf{2}$

2) $-(-8) - (7) + (-2) = 8 - 7 - 2$
   $= 1 - 2$
   $= \mathbf{-1}$

EXERCISE ON PART 3: OPERATIONS WITH SIGNED NUMBERS

1) $-14 \bullet 7$	2) $-9 \bullet -2$	3) $-45 \div 9$
4) -6.3×2.5	5) $11 \div -2$	6) $-72 \div -8$
7) $18 - (-15) - (-5)$	8) $-44 + (\frac{3}{8}) - (6)$	9) $-(-8) + (-7) - (-\frac{1}{2})$
10) $(-14) \bullet (-3) \bullet (6)$	11) $-(-2) + 8 - 7 - (-24) + (-7)$	12) $84 + (-99) + 44 - (-18) - 43$

PART 4: EXPONENTS

Before looking at ORDER OF OPERATIONS, it is necessary to understand how exponents function. Consider the following:

5^3

3 is the exponent

5 is the base

This is read 5 to the third power, and it means: $5 \times 5 \times 5 = 125$
As an introduction to algebra, the same is done with letters: $Y \bullet Y \bullet Y = Y^3$

Special case : Anything to the zero power is equal to 1.

a) $(3)^0 = 1$ b) $(-3)^0 = 1$ c) $x^0 = 1$ d) $(-x)^0 = 1$

Examples:

1) $(-3)^3 = -27$ 2) $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ 3) $(5x)^3 = 125x^3$

Note $\longrightarrow$ 4) $\mathbf{-2^2 = -(2 \times 2) = -4}$ but $\mathbf{(-2)^2 = -2 \times -2 = +4}$

EXERCISE ON PART 4: EXPONENTS

Evaluate or write without brackets.

1) 4^2	2) 1^5	3) $(-1)^6$	4) $(-2)^4$	5) $(x)^3$	6) $(6x)^2$
7) $\left(\frac{1}{3}\right)^2$	8) $\frac{5^2}{8}$	9) $(4y)^3$	10) $(-2xy)^3$	11) $-(-3z)^2$	

PART 5: ORDER OF OPERATIONS

When different operations are combined in one problem, they must be done in a certain order. The acronym for remembering the order of operations is **BEMA or BEDMAS.**

B - **B**rackets**.** All operations inside brackets must be done first.
For example: 2(3 + 5) = 2(**8**) = 16.

E - **E**xponents**.** An exponent indicates how many times to multiply a number by itself. Any exponents must be carried out next.
For example: $2(3^2 + 5) = 2(\mathbf{9} + 5) = 2\mathbf{(14)} = 28$

M - **M**ultiplication and **D**ivision must be carried out before adding and subtracting.
For example: 2 • 3 + 4 ÷ 2 = **6** + **2** = 8

A - **A**dding and **S**ubtracting are done last, as in the previous example.
For example: 6 + 3 • 5 -1 = 6 + **15** –1 = **21** – 1= 20

Fill in the missing steps to this example:

$$\frac{-2\,(10-6^2)}{3^2 \bullet 3^2 - 1} = \frac{-2\,(10 - \quad)}{\quad \bullet \quad - 1}$$

$$= \frac{-2\,(- \quad 6 \;)}{\quad - 1}$$

$$= \frac{13}{20}$$

EXERCISE ON PART 5: ORDER OF OPERATIONS

1) (8 – 2)(3 – 9)	2) $(-1)^4 + 2^3 - 10$	3) 8 – (2 • 3 – 9)
4) $-7(3^4) + 18$	5) 6 [9 – (3 – 4)]	6) 4 • 5 – 2 • 6 + 4
7) 9 ÷ (–3) + 16 • (-2) – 1	8) $\dfrac{5^2 - 4^3}{9^2 - 2^2}$	9) $\dfrac{(3-5)^2 - (7-13)}{(12-9)^2 + (11-14)}$
10) $[-12(-3) - 2^3] - (-9)(-10)$	11) $-2(16) - [\,2(-8) - 5^3\,]$	12) $3(-4.5) + (2^2 - 3 \bullet 4^2)$

PART 6: THE THREE RULES OF EXPONENTS

Rule 1: WHEN MULTIPLYING WITH THE SAME BASE, ADD THE POWERS.

1) $x^2 \bullet x^3 = x^{2+3} = x^5$

2) $4^2 \bullet 4^3 = 4^{2+3} = 4^5$

3) $a \bullet a^4 \bullet a = a^{1+4+1} = a^6$

4) $-y \bullet y^3 = -y^{1+3} = -y^4$

NOTICE, EVEN THOUGH THE EXPONENT OF 1 IS NOT WRITTEN, IT IS STILL THERE AND MUST BE ADDED.

When multiplying two terms such as $2x^2$ and $5x^4$, multiply the numbers and add the powers.

5) $(2x^2)(5x^4) = 10x^{2+4} = 10x^6$

6) $(-3ab^2)(-4a^2b) = 12a^3b^3$

Rule 2: WHEN DIVIDING WITH THE SAME BASE, SUBTRACT THE EXPONENTS

1) $\frac{2^5}{2^3} = 2^{5-3} = 2^2$ or 4

2) $\frac{a^8}{a} = a^{8-1} = a^7$

3) $\frac{b^4}{b^4} = b^0$ or 1

4) $\frac{-20x^5a^2}{5x^2a} = -4x^3a$

Rule 3: WHEN A POWER IS OUTSIDE BRACKETS, THE POWERS ARE MULTIPLIED

1.) $(c^2)^3 = c^{2 \bullet 3} = c^6$

2.) $(3x^3)^2 = 9x^6$

3.) $(5x^2y^4)^2 = 25x^4y^8$

EXERCISE ON PART 6: USING EXPONENTS

1) $2^2 \bullet 2^5$

2) $x^2 \bullet x \bullet x^3$

3) $2a \bullet 5a^3$

4) $3z^2 \bullet -4z^5$

5) $\frac{z^{12}}{z^3}$

6) $\frac{-6a^7z^2}{3a^3z}$

7) $(-7ab^3)(-3b)$

8) $a \bullet a^3 \bullet a^2$

9) $(m^3)^7$

10) $(3x^5)^3$

11) $(-3xy)(-6x^3)$

12) $\frac{-25xy^5}{-10xy}$

13) $y^3 \bullet (5x^4y)^2$

14) $\frac{x \bullet (2x^4y^2)^3}{x^2 \bullet 3xy^3}$

15) $-(-3a^5b)^3 + (-2a^{15}b^3)$

PART 7: SOLVING EQUATIONS - *ADDITION AND SUBTRACTION*

An equation is solved when the unknown letter is isolated on one side of the equal sign. When isolating x, the equation must be kept balanced. To maintain balance, you must always do the same thing to both sides of the equation.

For example: Solve for x: $x + 6 = 32$ → to isolate x, we must subtract 6 from the left, and thus from the right.

$$x + 6 - \mathbf{6} = 32 - \mathbf{6}$$
$$x = 26$$

Another example:

$$45 = -12 - x$$
$$45 + 12 = -x$$
$$57 = -x$$
$$-57 = x$$

[**Note**: we want to solve for +x, not –x]

EXERCISE ON SECTION 7: EQUATIONS WITH ADDITION AND SUBTRACTION

Solve for the missing letter.

1) $z - 3 = 25$

2) $a + 6.5 = 0.009$

3) $-34 = -6 - y$

4) $\frac{-3}{20} = y - 6$

5) $(-x) + \frac{4}{7} = -\frac{1}{3}$

6) $-9.65 = 0.8 - x$

7) $436 = a - 58$

8) $-9.6 - x + 3.4 = \frac{1}{2} - 3$

9) $-(6 + x) = \frac{2}{3} + (-7)$

10) $\frac{3}{4} - \frac{7}{8} = x + 0.9$

PART 8: SOLVING EQUATIONS - *MULTIPLICATION AND DIVISION*

These types of equations always have a number greater than 1 in front of the letter, or unknown. In the previous type of equation all you had to do was get your letter on one side of the equals sign and all the numbers on the other, and you were done! This is always your first step! Now, if your coefficient is greater than 1, you must divide both sides of the equal sign by the number in front of the letter. Here are three examples of how to tell what the coefficient is:

e.g. $-3x$ — coefficient = (-3)

e.g. $\frac{3x}{5}$ — coefficient = $\left(\frac{3}{5}\right)$

e.g. $\frac{x}{4}$ — coefficient = $\left(\frac{1}{4}\right)$

In this section, we will just concentrate on the second step for solving equations. In the next section, we will combine both steps.

1) $6z = -9$

$$\frac{6z}{6} = -\frac{9}{6}$$

$$z = -\left(\frac{3}{2}\right)$$

2) $\frac{1}{3}x = 5$

$$\frac{3}{1} \bullet \frac{1}{3}x = 5 \bullet \frac{3}{1}$$

$$x = 15$$

3) $\frac{x}{5} = \frac{4}{5}$

$$\frac{5}{1} \bullet \frac{x}{5} = \frac{4}{5} \bullet \frac{5}{1}$$

$$x = 4$$

EXERCISE ON PART 8: EQUATIONS WITH MULTIPLICATION AND DIVISION

1) $2x = -16$

2) $-4y = -35$

3) $-86 = 5z$

4) $\frac{x}{2} = \frac{3}{5}$

5) $2.56b = -1.28$

6) $\frac{2}{3}x = 5$

7) $-\frac{1}{3}t = 7$

8) $50 = -x$

9) $-\frac{2r}{3} = -\frac{27}{4}$

10) $\frac{x}{-5} = (-12.06)$

PART **9: SOLVING EQUATIONS** – *BOTH METHODS TOGETHER*

Before we look at the combination of both methods, we must first review adding and subtracting **like terms.** When adding terms that have the **same letter and same exponent, add the coefficients and carry the letter.**

e.g. $2x + 3x = 5x$ e.g. $6x - x = 5x$ e.g. $-4y - 3y = -7y$

TWO STEPS TO SOLVE EQUATIONS:

1.) COLLECT ALL NUMBERS ON ONE SIDE OF THE EQUAL SIGN, AND COLLECT ALL LETTERS ON THE OTHER SIDE.
2.) SIMPLIFY AND DIVIDE BY THE NUMBER IN FRONT OF THE LETTER.

EXERCISE ON PART 9: EQUATIONS WITH BOTH PRINCIPLES.

1) $5x + 6 = 31$ 2) $8x + 4 = 68$ 3) $-5y - 7 = 108$

1) Solve:

$$\begin{aligned} 3x + 4 &= 13 \\ 3x &= 13 - 4 \\ 3x &= 9 \\ \frac{3x}{3} &= \frac{9}{3} \\ x &= 3 \end{aligned}$$

2)

$$\begin{aligned} 2x - 2 &= -3x + 3 \\ 2x - 2 + 2 &= -3x + 3 + 2 \\ 2x &= -3x + 5 \\ 2x + 3x &= -3x + 3x + 5 \\ 5x &= 5 \\ \frac{5x}{5} &= \frac{5}{5} \\ x &= 1 \end{aligned}$$

4) $-91 = 9t + 8$ 5) $5x + 7x = 72$ 6) $-4y - 8y = 48$

7) $x + \frac{1}{3}x = 8$ 8) $5x + 3 = 2x + 15$ 9) $2x - 1 = 4 + x$

10) $4 + 3x - 6 = 3x + 2 - x$ 11) $5y - 7 + y = 7y + 21 - 5y$

PART 10: INTRODUCTION TO FACTORING

Key Concept:	In algebra, when we refer to a **term** it means to add and subtract to find a sum. In algebra, when we refer to a ***factor*** in algebra it means to multiply to find a product.

Factoring is a process for changing a term or group of terms into a multiplication.

A number has factors, and the factors for the number 20 are:
1 and 20; 2 and 10, and 4 and 5 because 1 * 20 = 20 and 2 * 10 = 20 and 4 * 5 = 20.

Algebraic expressions or groups of terms that contain numbers and letters can also be '***factored***'.

1) ***Common Factoring***: Always look for a common factor when factoring algebraic expressions

a) Common factoring can involve a common number (the highest number which divides evenly into every **term** in the expression. Example: $4x – 8 = 4 (x - 2)$ note the number 4 is common to both terms

b) Common factoring can involve a common letter (a letter that appears in every **term** in the expression
Example: $7x - 5xy = x (7 – 5y)$ note that the letter x is common to both terms

c) Common factoring can involve common letter, common letters and exponents (see section 4)
Example: $2x^5 – 4x^3 = 2x^3 (x^2 – 2)$ note that number and letters are common, <u>and the lowest power (exponent) to appear is its common *factor*</u>

Try these:

1) $6x – 36$ 2) $2a – 60$ 3) $ab + ac$ 4) $16z – 25y$ 5) $x^5 – 6x^2$

6) $10x^2y – 20xy + 30y$

PART 11: FACTORING – DIFERENCE OF SQUARES

A square is a number, or letter (variable) multiplied by itself. Here are some examples of ‘perfect’ squares:

$3^2 = 3 * 3 = 9$ $11^2 = 11 * 11 = 121$ $x^2 = x * x$ $a^2b^2 = ab * ab$

Here are two examples of difference of squares, and you will notice that the squares are subtracted from one another. This is why this type of factoring is called the difference of squares.

$x^2 – 25 = (\quad)(\quad) =$ (always set up your brackets first)

$(x – 5)(x + 5)$ or $(x + 5)(x - 5)$
(Note: the order of your brackets does not matter but one bracket always contains + and the other bracket contains -)

$y^2 – 121 = (\quad)(\quad) = (y - 11)(y + 11)$ or $(y + 11)(y - 11)$

When factoring a difference of squares, always look for a common factor first:

$4x^2 – 100 = 4(x^2 – 25) = 4(x - 5)(x + 5)$

Try factoring these:

1) $x^2 – 1$ 2) $a^2 – b^2$ 3) $x^2 + 9$ 4) $9a^2 – 25y^2$ 5) $x^2y^2 – 4$

6) $a^4 – 36$

PART 12: FACTORING A BASIC TRINOMIAL

The multiplications below produce trinomials (three terms) with an **x^2 term**, an **x term**, and a **number term**. For example:

1) $(x + 5)(x + 2)$ 2) $(x – 3)(x - 2)$ 3) $(x – 5)(x + 3)$ (FOIL)
$= x^2 + 2x + 5x + 10$ $= x^2 – 2x – 3x + 6$ $= x^2 + 3x – 5x – 15$ (collect like terms) $= x^2 + 7x + 10$ $= x^2 – 5x + 6$
$= x^2 – 2x – 15$

Steps to factor a basic trinomial:

a) Set up two brackets with the appropriate letter ie. (x) (x) or (b) (b)

b) Decide which signs go in the brackets. If the number term is positive the numbers to multiply will be either – both positive if x term is positive (see example 1), and both negative if the x term is negative (see example 2). If the number term is negative, the numbers which multiply must have opposite signs (see example 3)

c) The number which goes in the brackets must multiply to be the number term and add up to be the coefficient of the x term.

1) $x^2 + 3x + 2$ = (x +) (x +) $= (x + 2)(x + 1)$

2) $x^2 – 3x + 2$ = (x -) (x -) $= (x – 2)(x – 1)$

3) $x^2 + 3x – 10$ = (x +) (x -) $= (x + 5)(x - 2)$ or $(x – 2)(x + 5)$

4) $x^2 – 3x – 10$ = (x -) (x +) $= (x – 5)(x + 2)$ or $(x + 2)(x – 5)$

When you factor trinomials, always look for a common factor first. Try factoring these trinomials:

1) $x^2 + 15x + 50$ 2) $x^2 – 12x + 20$ 3) $y^2 + 4y – 12$ 4) $z^2 – 5z – 14$ 5) $2x^2 - 40x + 200$

ANSWERS PART 2

1) –7	2) –8	3) $-\frac{3}{8}$	4) –14	5) –10
6) –11	7) –45	8) –53	9) $-1\frac{5}{12}$	10) 0
11) –2.7	12) 3			

ANSWERS PART 3

1) –98	2) 18	3) –5	4) –15.75	5) –5.5
6) 9	7) 38	8) $-49\frac{5}{8}$	9) $1\frac{1}{2}$	10) 252
11) 20	12) 4			

ANSWERS PART 4

1) 16	2) 1	3) 1	4) 16	5) x^3
6) $36x^2$	7) $\frac{1}{9}$	8) $3\frac{1}{8}$	9) $64y^3$	10) $-8x^3y^3$
11) $-9z^2$				

ANSWERS PART 5

1) –36	2) –1	3) 11	4) –549	5) 60	6) 12
7) –36	8) $-\frac{39}{77}$	9) $\frac{5}{3}$ or $1\frac{2}{3}$	10) –62	11) 109	12) –57.5

ANSWERS PART 6

1) 2^7	2) x^6	3) $10a^4$	4) $-12z^7$	5) z^9	6) $-2a^4z$
7) $21ab^4$	8) a^6	9) m^{21}	10) $27x^{15}$	11) $18x^4y$	12) $2.5y^4$
13) $25x^8y^5$	14) $\frac{8}{3}x^{10}y^3$	15) $25a^{15}b^3$			

ANSWERS PART 7

1) 28	2) -6.491	3) 28	4) $5\frac{17}{20}$	5) $\frac{19}{21}$	6) 10.45
7) 494	8) -3.7	9) $\frac{1}{3}$	10) –1.025		

ANSWERS PART 8

1) -8 2) 8.75 3) -17.2 4) $1\frac{1}{5}$ 5) -0.5 6) $7\frac{1}{2}$

7) -21 8) -50 9) $10\frac{1}{8}$ 10) 60.3

ANSWERS PART 9

1) 5 2) 8 3) -23 4) -11 5) 6 6) -4

7) 6 8) 4 9) 5 10) 4 11) 7

ANSWERS PART 10

1) $6(x - 6)$ 2) $2(a - 30)$ 3) $a(b + c)$ 4) no common factor 5) $x^2(x^3 - 6)$ 6) $10y(x^2 - 2x + 3)$

ANSWERS PART 11

1) $(x + 1)(x - 1)$ 2) $(a - b)(a + b)$ 3) does not factor – sum of squares

4) $(3a - 5y)(3a + 5y)$ 5) $(xy - 2)(xy + 2)$ 6) $(a^2 + 6)(a^2 - 6)$

ANSWERS PART 12

1) $(x + 5)(x + 10)$ 2) $(x - 10)(x - 2)$ 3) $(y + 6)(y - 2)$ 4) $(z - 7)(z + 2)$

5) $2(x2 - 20 + 100) = 2(x - 10)(x - 10)$ (remember to look for a common factor first!)

ANSWERS PART 13 – please see instructor for answer to graphing question

APPENDIX 1

Equation of a Straight Line

The equation of a straight line is usually written this way: y = mx + b

What does it stand for? **y** = vertical or how far up **x** = horizontal or how far along

m = Slope or Gradient (how steep the line is) **b** = the Y Intercept (where the line crosses the Y axis)

With this example equation you can now choose any value for x and find the matching value for y

Y = x + 2

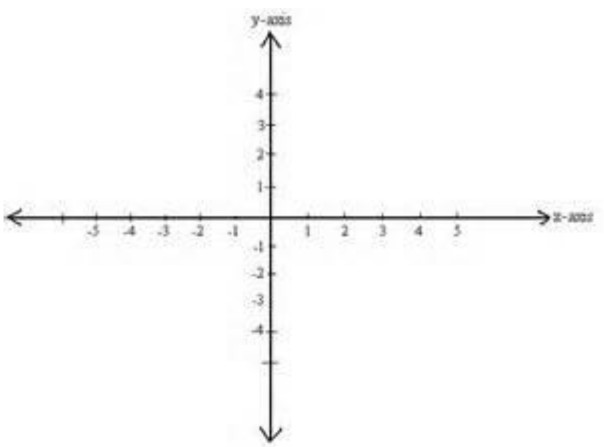
For example, when x is 2: y = 4 And so, when x = 2, y = 4

And, when x is 1: y = 3 And so, when x = 1, y = 3

And, when x is 0: y = 2 And so, when x = 0, y = 2

And, when x = -1 y = 1 And so, when x = -1, y = 1

Figure 1



The ordered pairs (x, y) for this line are (-1, 1) (0, 2) (1, 3) & (2, 4).
Try plotting these ordered pairs on Figure 1

For more practice and help with graphing lines try doing a google search for "equation of a line" or visit the Khan Academy at: www.**khanacademy**.org